EVALUATION OF PROJECTS FOR REHABILITATION OF HIGHWAY BRIDGES

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ABSTRACT: This paper presents a decision-support system for selecting the best set of strategies (projects) for bridge rehabilitation and replacement on a highway network, specifically during a long-range planning of the bridge funding needs for budgeting and legislative purposes. The proposed methodology recognizes the following aspects of the decision problem faced by the bridge engineer: (1) Multiple-attribute nature of bridge deficiencies; (2) risk impact of the predicted deterioration of the bridge; and (3) uncertainty due to the subjectively estimated decision variables. Based on a systems approach, the decision theory concepts of multiple criteria decision making (MCDM), utility theory, and decision making under risk (DMUR) are applied in formulating a decision-support system, with the estimates of the decision variables represented as fuzzy numbers. Modeling the predicted deteriorated state (condition) of the bridge as state fuzzy probability vectors, the feasible strategies for bridge rehabilitation and replacement are evaluated under each probable state of the bridge. The evaluation of projects at each bridge site is based on the possibility distributions of the expected fuzzy utilities, which are computed as a benefit index of each feasible strategy. An example application is also described to illustrate the application of this decision-support system.

INTRODUCTION

The seriously deteriorating condition of infrastructure, especially highway bridges, in the United States has generated considerable attention from engineers and public administrators. In a recent annual report of the Secretary of Transportation to the United States Congress (“The Status” 1989), the urgent need for an efficient way of managing the nation’s bridges, was emphasized. In this report, it was revealed that as of June 30, 1988, about 41% (238,537 bridges) out of the nation’s 577,710 inventoried bridges, were classified as either structurally deficient or functionally obsolete. A structurally deficient bridge, according to the definition of the Federal Highway Administration (FHWA), is a bridge that either has a restricted load-carrying capacity (cannot carry its original design load), has been closed, or requires immediate improvement to remain open (Recording 1979). A functionally obsolete bridge is one that may be structurally inadequate (current state legal load exceeds original design load), has an intolerable approach roadway geometry, narrow bridge-roadway width, or low underclearance or overclearance. An estimated $50 billion will be needed to bring all the nation’s bridges to an acceptable and safe standard (Hudson et al. 1987). From these statistics of deficient bridges and the current state of the national economy, it is obvious that there is a limited amount of funds available for the repair of these bridges; so, the selection of appropriate bridge repair strategies must be done in a realistic and effective manner.

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This paper describes a decision-support system for the evaluation of feasible strategies (projects) of bridge rehabilitation and replacement (improvement). By recognizing the various decision problems, and also allowing an incorporation of the decision maker’s human judgment and experience into the decision-making process, this system should provide bridge engineers and managers with an effective and realistic tool for evaluating alternative solutions to their bridge-management problems. In the following section, the existing methods employed by various transportation agencies in selecting bridge-improvement projects and allocating bridge funds—the bridge rehabilitation and replacement programs—is discussed, along with the shortcomings of these methods.

BRIDGE REHABILITATION AND REPLACEMENT PROGRAMS

The financing of bridge needs by most state transportation agencies is done as part of the planning/program formulation process (“Bridge” 1987). The planning activity is concerned primarily with distribution of funds; that is, decisions on usage of state or federal funds. The program formulation mainly involves project selection; that is, assessing each bridge in terms of its conditions, suggesting alternative improvement strategies based on an individual bridge’s deficiencies, and then selecting the best mix of improvement strategies (“Bridge” 1987). States’ procedures for selection of bridge-improvement projects can be categorized into three basic methods (“Bridge” 1987). The first step in each of the three methods is the initial screening of potential bridge-improvement projects using criteria such as the FHWA’s sufficiency rating. The first approach used in the selection of projects is based completely on the intuition and expertise of the bridge managers. In the second approach the bridge-improvement alternatives are investigated and other priority indices such as cost, deficiency points, and detour length are used to select projects; this approach is labeled the priority-ranking method. The third method is similar to the second one except that a network of bridge-improvement alternatives is considered (network level): a benefit index is assigned to the improvement project at each bridge site, and the set of projects with maximum benefits is selected through the use of some mathematical programming or optimization techniques.

Existing methodologies do not adequately account for the uncertainties involved in the decision process, particularly those of imprecision or vagueness in variables that are subjectively estimated. Also, literature review and interviews with state transportation agencies have revealed that a lot of human judgment in the form of expert opinion goes into many decisions related to bridge management (“Bridge” 1984, 1987; Stukhart et al. 1989). It is possible to select a wrong alternative strategy because of the imprecise parameters used in the decision process, giving a false sense of accuracy in the results. Expert opinion is usually needed to realistically estimate bridge deterioration rates, to measure the bridge deficiency, to estimate the cost of rehabilitation, and also to estimate the benefits of particular rehabilitation efforts. The imprecision in these variables needs to be quantified, and its effects reflected in the final outcome of the decision analysis.

In addition to the shortcoming just mentioned, most of existing bridge rehabilitation project-selection methods do not properly incorporate the stochastic deterioration process (probabilistic estimates) of a bridge into the decision analysis. Shirole and Hill (1987a, b) proposed a systems approach in the form of statistical decision trees, but failed to consider the bridge-deterioration pattern or account for the imprecision in the estimates of the
decision variables. Saito and Sinha (1989) also applied a systems approach, incorporating a Markovian (stochastic) form of bridge-deterioration model in the optimization module of their decision model; but they only used it to update the bridge condition in the algorithm of the multiperiod decision process. The shortcomings just mentioned can be corrected by doing the following:

1. Devise a systematic approach to consider a prescribed set of feasible alternative bridge-improvement projects at each bridge site both at the project and network levels of decision making instead of just an already selected improvement project.
2. Consider the benefits associated with these prescribed alternatives, instead of the deficiencies of the existing bridge.
3. Account for the imprecision or uncertainties involved in the decision variables, especially those that are subjectively estimated by the bridge engineer.
4. Incorporate the bridge-deterioration process into the decision analysis in order to make long-range planning decisions.

THEORETICAL BACKGROUND OF PROJECT EVALUATION METHODOLOGY

A systems approach framework is applied to formalize the decision-making process just described through the integration of three concepts in decision theories-multiple criteria decision making (MCDM), utility theory, and decision making under risk (DMUR)-to formulate a methodology for realistically selecting the best bridge-rehabilitation and replacement strategies. In this framework, fuzzy-set theoretic mathematics is used to account for the imprecision, vagueness, or “fuzziness” of variables involved in the decision-making process.

Multiple Criteria Modeling

The multiple criteria environment of decision making in the management of highway systems has been recognized early as is evident in some existing decision-support methods such as the level-of-service approach in highway maintenance management (Kulkarni and Van til 1984), and the level-of-service approach in bridge-management activities (Chen and Johnston 1987). Most state transportation agencies also use priority-ranking formulas based on a consideration of multiple criteria. The benefit associated with each alternative bridge-improvement project can be realistically estimated by evaluating the effectiveness of this improvement effort in terms of the deficiencies to be corrected (or expected to be corrected) on the existing structure. By classifying the various deficiencies on the structure in terms of the bridge attributes-deck width, load capacity, vertical clearance, etc.—the multifacet nature of the benefits associated with correcting the deficiencies thus puts this type of decision making in the class of a multiple criteria decision making (MCDM).

Because of the human judgment involved in an MCDM environment, the preferences of a decision maker (DM) can be modeled using the fuzzy binary relations approach (Blin and Whinston 1973; Klir and Folger 1988) or the maximum expected utility approach (Ang and Tang 1984). The former involves a complicated pairwise comparison of bridge-improvement alter-
natives under the various decision criteria, a process that may not be prac-
tically feasible when applied to the type of decision problem addressed in
this paper. On the other hand, the maximum-expected-utility approach is
more practical, employing utility functions to model the decision maker’s
preferences in terms of the criteria being considered.

Maximum Expected Utility Approach
Utility theory is simply a method for assessing the worth a DM attaches
to a particular value during a decision-making process. This value represents
one of the possible outcomes a particular alternative may have when eval-
uated under some established criteria. For an alternative \( a_i \), if the possible
outcome can be described by a utility \( u_{ij} \), then the expected utility of alter-
native \( a_i \) is given by

\[
E(U_i) = \sum_j p_{ij} u_{ij}; \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \ldots \ldots \ldots \ldots (1)
\]

where \( U_i \) = overall utility of the alternative \( a_i \); \( u_{ij} \) = utility of alternative
\( a_i \), under the \( j \)th consequence; \( p_{ij} \) = corresponding probability; \( m \) = number
of alternatives; and \( n \) = number of possible consequences.

Under the maximum expected utility approach, the decision criteria need
to be identified, and utility functions developed for each criterion to model
the DM’s value system. In Sobanjo (1991), these two tasks were accom-
plished based on the expert opinions of bridge engineers and the available
bridge data. Using Saaty’s (1980) analytical hierarchy process (AHP) to
stratify the objective of selecting bridge-improvement projects, eight
decision criteria were identified in the following three major categories: (1)
The ratio of the average daily traffic (ADT) to the cost of improvement
project; (2) improvement in the level-of-service criteria (structural condi-
tion, deck geometry, load capacity, waterway adequacy, approach align-
ment, and roadway clearance); and (3) the expected extension in bridge
service life. Utility functions were developed for each of these eight decision
criteria.

Concept of Fuzzy Number
Consider a set \( A \) with elements denoted by \( x \). Under the conventional
set theory, a characteristic or membership grade function \( \mu_A \) can be used
to define the membership of any element or subset in the set \( A \) as follows:

\[
\mu_A(x) = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{otherwise} 
\end{cases} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)
\]

Instead of the \( \{0, 1\} \) (yes or no) valuation, if the membership grade \( \mu_A(x) \)
can have values in the real interval \([0, 1] \) according to how much \( x \) belongs
to this set \( A \), then the set \( A \) is a fuzzy set (Zadeh 1965). If the set \( A \) is a
set of criteria, \( \mu_A(x) \) is the degree to which \( x \) satisfies the conditions of \( A \),
or, in other words, \( \mu_A(x) \) is the “strength” or “truth value” of the statement,
“\( x \) belongs to the set \( A \).” In fuzzy-set terminology, values that are known
precisely are referred to as “crisp ordinary numbers”; imprecise values are
represented by fuzzy subsets. In the methodology proposed in this paper,
the fuzzy-set concept is applied to deal with the imprecision in quantitative
values that are subjectively estimated by the bridge engineer. The bridge
engineer has to estimate the agency costs of bridge-improvement projects
based partly on engineering judgment, expertise, and experience. The en-
engineer also has to estimate the corresponding benefits associated with each feasible improvement project alternative.

In this paper, decision variables are represented as fuzzy numbers; algebraic computations are based on the interval of confidence or interval arithmetics described by Kaufmann and Gupta (1985). Under this approach, the a-cut or intervals of confidence of the fuzzy numbers are used to perform the necessary algebraic operations. In a general form of a fuzzy subset $A_a$, $a$ is the degree of belief or, in terms of fuzzy sets, the degree of membership; $A_a$ is the interval of confidence associated with the $a$ [formally referred to as the $a$-cut (Fig. 1)]. All possible values of $A$ whose degrees of belief are greater than or equal to a specified value $a$ constitute the $a$-cut, $A_a$, such that

$$A_a = \{x | \mu_A(x) \geq a\} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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In the proposed methodology, it is necessary to make decisions based on the ranking or comparison of utilities of bridge-improvement alternatives, utilities that are of the form of TFNs. The comparison of TFNs is mathematically done using two approaches: (1) A linear ordering or ranking of the fuzzy numbers based on an equivalent crisp ordinary number; and (2) a “qualified comparison” approach in which the “strength” or “truth values” of the resulting decisions are indicated by the a-cuts of the fuzzy utilities.

Kaufmann and Gupta (1988) discussed the linear ordering of fuzzy numbers using an index called the “removal” or “ordinary representative” of each fuzzy number—the crisp ordinary number equivalent of a fuzzy number. The “ordinary representative” (ORD) of a TFN $A = [l, m, h]$ can be computed as

$$A'' = \frac{(l + 2m + h)}{4} \quad \ldots \quad (5)$$

The mode of a fuzzy number is the most possible value under the distribution. Thus, for the TFN $A$, the mode is $m$, while the divergence or support is $(h - l)$.

The second approach is the “qualified comparison” approach. To compare two TFNs $A$ and $B$, a “qualified” statement can be made about the relative value of the parameters represented by these fuzzy numbers. If $A$ and $B$ denotes the expected fuzzy utilities of bridge-improvement strategies such as limited rehabilitation and replacement, respectively (Fig. 3), then by comparing $A$ and $B$ a “truth value” (Watson et al. 1979; Whalen 1987) can be attached to a statement as to whether limited rehabilitation is better than replacement of the entire bridge or vice versa.

By studying the possible values whose degrees of membership ($\alpha$) are greater than 0.8 (o-cut at 0.8), it could be seen that the lowest possible utility for strategy B (replace bridge) is higher than the highest possible utility for strategy A (limited rehabilitation). Using this standard of possibility $(a = 0.8)$, it could be said that the utility of B is strictly greater than the utility of alternative A. The “strength” of this statement or its “truth value” is given by the compliment of the lowest degree of membership (cr) above which the statement is true (Whalen 1987). Thus, the statement
“Replacing the bridge is strictly better than limited rehabilitation” has a “truth value” of $1 = 0.8$, or $0.2$.

Above the a-cut of $0.4$, an overlap occurs between possible utility values of these two strategies. The highest possible utility for strategy $B$ is still always higher than the highest possible utility for $A$, and the lowest possible utility for $B$ is still higher than the lowest possible utility for $A$, but the lowest utilities of $B$ are not higher than the highest utilities of $A$. Thus, the statement “Replacing the bridge is at least as good as limited rehabilitation” has a truth value of $1 = 0.4$, or $0.6$.

This qualified comparison method can be applied for a detailed analysis and selection between any pair of feasible bridge-improvement strategies. The truth value just mentioned can be computed from the graphical relationship of the TFNs. Consider any two TFNs $A$ and $B$, where $A = [l_1, m_1, h_1]$ and $B = [l_2, m_2, h_2]$. If $m_2 > m_1$, then the truth value of strict dominance as just discussed can be derived from the possibility level $t$, at which left reference function of $B$ intersects with the right reference function of $A$ (Fig. 3). Above this level $t$, all possible values of $B$ are greater than all possible values of $A$. The desired truth value is simply the complement of $t$. Looking at Fig. 3, the utility (x-coordinate) of the intersection point, say $x'$, can be computed as follows:

$$x' = \frac{h_1(m_2 - l_2) + l_2(h_1 - m_1)}{(h_1 - m_1) + (m_2 - l_2)}$$  \hspace{1cm} (6)$$

Then

$$t = \frac{h_1 - x'}{h_1 - m_1}$$  \hspace{1cm} (7)$$

Therefore, the truth value of strict dominance, $\alpha'$, can be computed as

$$\alpha' = 1 - t = \frac{1}{h_1 - m_1} \left[ \frac{h_1m_2 - m_1l_2}{(h_1 - m_1) + (m_2 - l_2)} - m_1 \right]$$  \hspace{1cm} (8)$$

FIG. 3. Possibility Analysis Profile Using Fuzzy Utilities
Decision Making under Risk (DMUR)

Because of the stochastic nature of the bridge deterioration process, any decision made on the bridge on a long-term or multiperiod basis may be classified a DMUR. DMUR is a framework in which the only available knowledge about the outcome states is the probability distribution (Bradley 1976). To model the bridge deterioration process, a probabilistic estimate of the expected bridge condition with respect to time can be determined based both on existing bridge-inspection records and the bridge engineer’s expert opinion.

Under this approach, the crisp probabilities may first be estimated using the classical statistical analysis of available historical data on the bridge. These crisp probabilities are then modified or replaced directly with fuzzy probabilities in which each of the possible probabilities can be assigned a membership grade (a measure of possibility) to obtain a possibility distribution for each probability estimate. Assuming a TFN for each distribution, the state probability vector can be modified to reflect the bridge engineer’s judgment on each state’s probability. That is,

\[
P_n = \left[ (p_{y_1}, p_{y_2}, p_{y_3}), (p_{s_1}, p_{s_2}, p_{s_3}), (p_{7_1}, p_{7_2}, p_{7_3}), \right] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
1. The set of feasible bridge-improvement alternatives is known.
2. The set of possible outcomes of each alternative under each criterion can be estimated by the bridge engineer, but as fuzzy numbers.
3. The states of nature (possible deteriorated states or condition of the bridge) are not known for sure, but the bridge engineer can identify each possible condition, and he or she can also assign a probability to the occurrence of each state. The engineer will provide information on the predicted future condition of the bridge in the form of a fuzzy state probability vector.

The objective of the bridge engineer is to select the best bridge-improvement project to be accomplished in the future on a particular bridge, employing the principle of maximum expected utility. The following steps will make up an algorithm that can be used to achieve this objective.

1. Declare the states of nature to be considered in the analysis; that is, possible deteriorated states or condition ratings \( s_k \) that the bridge or its component can be expected to be at a particular fixed age, where \( k = 1, 2, 3, \ldots, r \).
2. Synthesize feasible bridge-improvement alternative \( A_i \), and the cost \( C_i \), of the alternative, where \( i = 1, 2, 3, \ldots, m \), and \( C_i \) is of the form of a TFN triplet \((l, m, h)\).
3. Evaluate the score \( y_{ij} \), of bridge-improvement alternative \( A_i \), under each decision criterion \( c_j \), in the state \( s_k \), where \( j = 1, 2, 3, \ldots, n \), and \( y_{ij} \), is of the form of a TFN triplet \((l, m, h)\).

The decision matrix evaluated for \( m \) bridge-improvement alternatives under \( n \) criteria, at state \( k \), is shown in Fig. 4.

In Fig. 4, the left column represents the various feasible bridge-improvement alternatives, and top row represents the identification of the various decision criteria as follows:

- \( c_1 \) = ratio of the average daily traffic (ADT) to the project cost (ADT/Cost).
- \( c_2 \) = expected improvement in structural condition appraisal rating (Impstr).
- \( c_3 \) = expected improvement in deck geometry appraisal rating (Impdeck).
- \( c_4 \) = expected improvement in clearance appraisal rating (Impcle).
- \( c_5 \) = expected improvement in load capacity appraisal rating (Impload).
- \( c_6 \) = expected improvement in waterway adequacy appraisal rating (Impwat).
- \( c_7 \) = expected improvement in approach roadway alignment appraisal rating (Impappr).
- \( c_8 \) = expected extension in bridge service life (years) (EXTLIFE).

4. Determine the utility \( u_{ijk} \) of bridge-improvement alternative \( A_i \) under each decision criterion \( c_j \) in the state \( k \), where \( j = 1, 2, 3, \ldots, n \), and \( u_{ijk} \) is of the form of a TFN triplet \((l, m, h)\).
5. Compute the weighted utility \( U_{ijk} \) for each bridge-improvement alternative \( A_i \), under each decision criterion \( c_j \) in the state \( s_k \), where \( j = 1, 2, 3, \ldots, n \).

Let \( w_i = \) relative weight of decision criterion \( c_i \), such that \( w = [w_1, w_2, \ldots, w_n] \), and \( \sum_i w_i = 1 \). Therefore

\[
U_{ijk} = w_i u_{ijk}
\]

\((11)\)
TFNs $A = [l_1, m_1, h_1]$ and $B = [l_2, m_2, h_2]$, the multiplication operation can be represented as

$$A \times B = [l_1l_2, m_1m_2, h_1h_2].$$

**EXAMPLE APPLICATION**

Based on the framework and algorithms described earlier in this paper, a computer program was written in the C language to demonstrate the application of the utility-based decision-support methodology (Sobanjo 1991). The major characteristics of a typical deficient bridge are presented, along with feasible alternatives and their expected effectiveness in correcting these bridge deficiencies in terms of the eight decision criteria identified earlier.

Using a sample bridge data similar to the “Bridge Inventory, Inspection, and Appraisal Program (BRINSAP)” database of the Texas State Department of Highways and Public Transportation (SDHPT) (“Bridge” 1984), a problem case is created with a set of five feasible improvement alternatives, as listed in Table 1. Considering the decision variables described in the previous section of this paper, the estimates of the outcomes of each alternative are given as a TFN in the format of a triplet (minimum, most likely, maximum).

Assuming there are three probable states, or conditions, ($s_k$) that the bridge is expected to be in 10 years time; that is

$$k = 3, \text{ and } s_k = \{\text{"GOOD," "FAIR," "VERY POOR"} \}.$$

let the respective fuzzy probability vectors be represented by the state probability vector

$$p(s_k) = \begin{bmatrix} 0.1, 0.2, 0.41, 0.0, 0.2, 0.3 \end{bmatrix}.$$

The expected decision matrix under each of these three states will contain the evaluation of the bridge-improvement alternatives under each decision criterion, as shown in Figs. 5-7. The corresponding utility matrices based on utility functions developed in Sobanjo (1991) are shown in Figs. 8-10.

Let the set of relative criteria weights be given by

$$w^T = [0.1, 0.2, 0.05, 0.05, 0.2, 0.05, 0.15, 0.2].$$

Then, the weighted utility vectors can be computed as follows:

For state 1: $s_1 =$ “GOOD” condition of bridge

$$U^T = \begin{bmatrix} 16.5, 18.4, 19.81, [47.2, 51.9, 56.31, [64.9, 66.0, 67.91, [42.4, 49.8, 57.1], [69.6, 74.3, 76.1]] \end{bmatrix}.$$

For state 2: $s_2 =$ “FAIR” condition of bridge

$$U^T = \begin{bmatrix} 16.5, 18.4, 19.81, [47.2, 51.9, 56.31, [64.9, 66.0, 67.91, [42.4, 49.8, 57.11, [69.6, 74.3, 76.11]] \end{bmatrix}.$$

For state 3: $s_3 =$ “VERY POOR” condition of bridge

$$U^T = \begin{bmatrix} 16.5, 18.4, 19.81, [53.8, 58.3, 60.31, [64.9, 66.0, 67.91, [42.4, 49.8, 57.11, [76.8, 78.5, 79.91]] \end{bmatrix}.$$

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<table>
<thead>
<tr>
<th>Feasible bridge improvement alternative (1)</th>
<th>Project description (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rehabilitation I (A&lt;sub&gt;1&lt;/sub&gt;)</td>
<td>Limited rehabilitation of the bridge. Initial cost (in units of $10,000) is $[12, 15, 21]$, and the bridge service life is expected to be extended by $[5, 8, 12]$ years. Possible improvement in the level of service criteria are as follows: clearance appraisal rating by $[1, 1, 1]$; and waterway adequacy appraisal rating by $[1, 1, 1]$.</td>
</tr>
<tr>
<td>Rehabilitation II (A&lt;sub&gt;2&lt;/sub&gt;)</td>
<td>Rehabilitate the bridge deck and the superstructure. After the rehabilitation, the deck geometry is expected to improve by $[2, 2, 2]$ ratings, the structural condition by $[2, 3, 4]$ ratings, the load capacity by $[3, 3, 3]$ ratings, and the bridge service life may be extended by $[25, 35, 45]$ years. The estimated cost (in units of $10,000) of the rehabilitation is $[56, 59, 63]$.</td>
</tr>
<tr>
<td>Rehabilitation III (A&lt;sub&gt;3&lt;/sub&gt;)</td>
<td>Rehabilitate the bridge deck and the superstructure. Reconstruct the approaches. Improvement in the level of service expected after the rehabilitation will include: the deck geometry by $[2, 2, 2]$ ratings; the structural condition by $[3, 3, 3]$ ratings; the load capacity by $[3, 3, 3]$ ratings; the approach roadway alignment by $[4, 4, 4]$ ratings; and the bridge service life may be extended by $[31, 36, 451]$ years. The estimated cost (in units of $10,000) of the rehabilitation is (66, 75, 79).</td>
</tr>
<tr>
<td>Rehabilitation IV (A&lt;sub&gt;4&lt;/sub&gt;)</td>
<td>Raise the bridge, and lengthen the structure. After the rehabilitation, the deck geometry is expected to improve by $[3, 4, 5]$ ratings, the structural condition by $[2, 2, 2]$ ratings, the load capacity by $[2, 3, 6]$ ratings, and the bridge service life may be extended by $[25, 35, 45]$ years. The estimated cost (in units of $10,000) of this rehabilitation alternative is $[66, 74, 87]$.</td>
</tr>
<tr>
<td>Replacement (A&lt;sub&gt;r&lt;/sub&gt;)</td>
<td>Replace the existing structure with a new bridge. Improvement in the level of service expected due to the replacement will include: the deck geometry by $[6, 6, 6]$ ratings; the structural condition by $[5, 5, 5]$ ratings; the load capacity by $[4, 4, 4]$ ratings; the approach roadway alignment by $[3, 4, 5]$ ratings; and the bridge service life may be extended by $[45, 49, 54]$ years. The estimated cost (in units of $10,000) of this replacement alternative is $[76, 85, 90]$.</td>
</tr>
</tbody>
</table>

By multiplying the vectors of weighted utility with the state probability vector, the expected utility vector for each bridge-improvement alternative project is

$$U^T = \{[9.9, 18.4, 27.71, [28.3, 53.2, 80.01, [38.9, 66.0, 95.11], [25.4, 49.8, 79.91], [41.8, 75.1, 100]]\}.$$

For example, the second element of the last vector represents the fuzzy expected utility of the alternative $A_2$, computed as follows:

$$U^T_2 = \sum_k u_k \otimes p(s_k) = ([47.2, 51.9, 56.31 \otimes [0.1, 0.2, 0.41])$$
### TABLE 2. Preference Ranking Based on TFN Modes (Fuzzy Utility)

<table>
<thead>
<tr>
<th>Rank (1)</th>
<th>Alternative (2)</th>
<th>Expected fuzzy utility (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Replacement (Aₐ)</td>
<td>[41.8, 75.1, 100]</td>
</tr>
<tr>
<td>2</td>
<td>Rehabilitation III (Aₐ)</td>
<td>[38.9, 66.0, 95.1]</td>
</tr>
<tr>
<td>3</td>
<td>Rehabilitation II (A₂)</td>
<td>[28.3, 53.2, 80.01]</td>
</tr>
<tr>
<td>4</td>
<td>Rehabilitation IV (Aₐ)</td>
<td>(25.4, 49.8, 79.9)</td>
</tr>
<tr>
<td>5</td>
<td>Rehabilitation I (Aₐ)</td>
<td>[9.9, 18.4, 27.7]</td>
</tr>
</tbody>
</table>

### TABLE 3. Preference Ranking Based on TFN Modes (Fuzzy Utility Per Unit Cost)

<table>
<thead>
<tr>
<th>Rank (1)</th>
<th>Alternative (2)</th>
<th>Expected fuzzy Util/cost (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rehabilitation I (A₁)</td>
<td>[0.47, 1.23, 2.30]</td>
</tr>
<tr>
<td>2</td>
<td>Rehabilitation II (A₂)</td>
<td>[0.45, 0.90, 1.43]</td>
</tr>
<tr>
<td>3</td>
<td>Rehabilitation III (A₃)</td>
<td>[0.45, 0.89, 1.44]</td>
</tr>
<tr>
<td>4</td>
<td>Replacement (A₄)</td>
<td>[0.46, 0.88, 1.31]</td>
</tr>
<tr>
<td>5</td>
<td>Rehabilitation IV (A₄)</td>
<td>(0.32, 0.66, 1.21)</td>
</tr>
</tbody>
</table>

### TABLE 4. Preference Ranking Based on ORD (Fuzzy Utility)

<table>
<thead>
<tr>
<th>Rank (1)</th>
<th>Alternative (2)</th>
<th>Expected utility (ORD) (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Replacement (Aₐ)</td>
<td>73.0</td>
</tr>
<tr>
<td>2</td>
<td>Rehabilitation III (Aₐ)</td>
<td>66.5</td>
</tr>
<tr>
<td>3</td>
<td>Rehabilitation II (A₂)</td>
<td>53.7</td>
</tr>
<tr>
<td>4</td>
<td>Rehabilitation IV (A₄)</td>
<td>51.2</td>
</tr>
<tr>
<td>5</td>
<td>Rehabilitation I (Aₐ)</td>
<td>18.6</td>
</tr>
</tbody>
</table>

### TABLE 5. Preference Ranking Based on ORD (Fuzzy Utility Per Unit Cost)

<table>
<thead>
<tr>
<th>Rank (1)</th>
<th>Alternative (2)</th>
<th>Expected Util/cost ORD (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rehabilitation I (A₁)</td>
<td>1.31</td>
</tr>
<tr>
<td>2</td>
<td>Rehabilitation II (A₂)</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>Rehabilitation III (A₃)</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>Replacement (A₄)</td>
<td>0.88</td>
</tr>
<tr>
<td>5</td>
<td>Rehabilitation IV (A₄)</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The resulting preference rankings are as shown in Tables 2-5. Based on TFN modes (most likely values) of the expected fuzzy utilities, the best bridge-improvement project is Replacement (Table 2). The engineer’s preference ranking of bridge-improvement projects, based on the TFN modes of expected fuzzy utility per unit cost, is shown in Table 3. But in this case, the best bridge-improvement project is Rehabilitation I.

Preference ranking based on the “ordinary representative” (ORD) of expected fuzzy utilities indicates Replacement as the best bridge-improvement project (Table 4). On the other hand, Table 5 shows Rehabilitation I as the best project based on the TFN modes of the expected fuzzy utility per unit cost.
The “qualified comparison” of the alternatives can also be done based on either the expected fuzzy utility or the expected fuzzy utility per unit cost (Tables 6 and 7). The relative comparison is reflected in the truth values shown.

Applying (8) in Table 7, the alternative $A_1$ (Rehabilitation I) is “strictly better” than $A_2$ (Rehabilitation II) at a truth value of

$$
\alpha' = \frac{1}{1.43 - 0.90} \left( \frac{(1.43)(1.23) - (0.90)(0.47)}{(1.43 - 0.90) + (1.23 - 0.47)} - 0.9 \right) = 0.25 \cdot (34)
$$

It was shown in Sobanjo (1991) that the proposed utility-based methodology can be used to correct the shortcomings of existing decision-making algorithms such as the simple benefit-cost analysis, the incremental benefit-cost (INCBEN) technique (MacFarland et al. 1983a; Farid et al. 1988), and optimization models (Subramanian et al. 1983; MacFarland et al. 1983b; Sinha et al. 1989). The computed ordinary representative of the expected fuzzy utility of each bridge-improvement alternative, which is a single-valued measure of benefit, can be incorporated into these algorithms. Thus, the best bridge-improvement project can be selected based on the principle of maximum expected utility. It should be noted that the ranking and comparison of fuzzy utility per unit cost of project ($\text{Util}/\text{cost}$) would recommend the most cost-effective bridge-improvement alternative project. Ranking the projects by the fuzzy utility alone will most of the time suggest bridge replacement or expensive bridge rehabilitation projects as the best project. This is simply because a new bridge will usually contribute more to an improvement in the level-of-service attributes; that is, load capacity, deck geometry, and so on.
CONCLUSIONS

It was demonstrated in this paper that it is feasible to formulate a project-evaluation methodology for a long-range (multiperiod) planning of bridge funding needs that will realistically recognize the multiple-attribute nature of bridge deficiencies, and also quantify the effects of the uncertainties in the decision variables due to the predicted deterioration of the bridge and the subjectivity in estimating these variables. An expected fuzzy utility is computed as a benefit index for each feasible bridge-improvement project. This benefit index may be used to produce a preference ranking of the projects, or it may be incorporated into existing project-selection algorithms such as the simple benefit-cost analysis, the incremental benefit-cost (INC-BEN) technique, and mathematical optimization algorithms (integer and dynamic programming).

Utilizing the attractive features of a systems approach to problem solving, it is hoped that the framework and the algorithm of the methodology formulated in this paper will contribute to the development of a comprehensive bridge-management system in an effort to address some of the recent problems caused by the seriously deteriorated state of the highway bridges in the United States.

APPENDIX. REFERENCES


cremental benefit-cost technique.” *Tech. Rep. Prepared for Federal Highway Administration*, Texas Transportation Institute, Texas A&M University, College Station, Tex.


*Recording and coding guide for the structural inventory and appraisal guide of the nation’s bridges.* (1979). Federal Highway Administration (FHWA), Washington, D.C.


